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Consistent specification testing for conditional moment restrictions

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Abstract

This paper introduces specification tests for conditional moment restrictions. The proposed tests are generalizations of the Kolmogorov–Smirnov and Cramer–von Mises tests and they are consistent against all alternatives to the null hypothesis, powerful against $1/\sqrt{n}$ local alternatives and not dependent on any smoothing parameter. A nonparametric bootstrap procedure based on recentered criterion function is suggested to obtain critical values for the tests and is justified asymptotically.

Keywords: Bootstrap; Conditional moment restriction; Consistent specification test; GMM

JEL classification: C12

1. Introduction

This paper considers specification tests for conditional moment restrictions for independent observations. Let $\{W_i = (Y_i', X_i')' \in R^L \times R^K: i = 1, \ldots n\}$ denote the observed random sample. We are interested in testing the null hypothesis

$$H_0 : E\,[\rho(Y_i, \theta_0)|X_i] = 0 \text{ a.s. for some } \theta_0 \in \Theta \subset R^P \tag{1}$$

where $\rho(.,.) : R^L \times \Theta \rightarrow R$ is a known function. The alternative hypothesis is the negation of H_0, that is,

$$H_1 : E\,[\rho(Y_i, \theta)|X_i] \neq 0 \text{ with positive probability for all } \theta \in \Theta. \tag{2}$$

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(It is possible that some of the elements Y_i and X_i may overlap.) An important example of such restriction is one where $\rho(Y_i, \theta_0)$ is the error term of a nonlinear regression model. The restriction of the type (1) is very common in economics. For example, an Euler equation from an optimizing behavior of an economic agent under certain parametric assumption (e.g., on the utility function) often yields such restriction with X_i corresponding to the variables in the information set (see, e.g., Hansen and Singleton, 1982; Newey, 1990). In such case, our tests discussed below can be used to test validity of the parametric assumption imposed.

The tests we consider are generalizations of the Kolmogorov–Smirnov and Cramer–von Mises tests of goodness of fit. The tests introduced in this paper are (i) consistent against all alternatives to the null hypothesis H_0; (ii) powerful against $1/\sqrt{n}$ local alternatives to H_0; (iii) not dependent on any smoothing parameter; and (iv) simple to compute.

The testing problem considered here is related to the existing literature on consistent testing for parametric regression functions against nonparametric alternatives. Examples of such test include Ait-Sahalia et al. (2000); Bierens (1982, 1990); Bierens and Ploberger (1997); De Jong (1996); Eubank and Spiegelman (1990); Fan and Li (1996); Gozalo (1993); Härdle and Mammen (1993); Hong and White (1996); Li and Wang (1998); Whang (2000); Wooldridge (1992); Yatchew (1992); and Zheng (1996), to mention only a few. To the best of our knowledge, most of the existing results in the literature look into the special case in which $Y_i = (Y_i^*, X_i)$ and $\rho(Y_i, \theta) = Y_i^* - h(X_i, \theta)$ for some parametric function $h(.,.)$, where Y_i^*, is the dependent variable[1]. The existing methods, however, cannot be directly applicable to the general case in which the dependent variable is related to the regressors through the implicit function $\rho(. , .)$. Because of this general nature of ρ, we use a nonparametric bootstrap procedure with recentered criterion function to approximate the asymptotic null distribution of our test statistics.

Our tests are also related to the test of overidentifying restrictions in the GMM framework, which tests the validity of unconditional moment restriction, e.g., $Ef(W_i, \theta_0) = 0$, where $f(W_i, \theta) = \rho(Y_i, \theta)A(X_i)$ and $A(.) : R^K \to R^M$ is a measurable function. However, since $A(.)$ is a finite-dimensional function, the latter test is not consistent against all the alternatives of the form (2).

2. The asymptotic null distributions

Let $g(W_i, \theta, x) = p(Y_i, \theta)(X_i \le x)$, where $(X_i \le x)$ denotes the indicator function of the event $X_i \le x$.[2] To motivate our tests, note that, under the null hypothesis, we have $E\ [g(W_i, \theta_0, x)] = 0\ \forall x \in R^K$, for

[1] We note that Lewbel (1995) develops a consistent test of general conditional moment restrictions in which the moment function depends on data through a nonparametric function and applies it to test for Slutsky symmetry of demand system without imposing any parametric assumptions. The focus of our test, however, is slightly different from his since we are mainly interested in testing the validity of parametric specification of the moment function. On the other hand, Lewbel (1995) considers a Bierens (1990) type test statistic using an exponential function as a weighting function. However, the choice of this weighting function requires one to choose an nuisance space T^* over which one calculates the test statistic. But, in practice, testing results could be sensitive to both the choice of T^* and the method of grid search over T^* to calculate the test statistic. On the other hand, the indicator function which we use as our weighting function does not have such problems.

[2] In the multivariate case $K > 1$, where $X_i = (X_{i1}, \ldots, X_{iK})'$ and $x = (x_1, \ldots, x_K)'$, we define $(X_i \le x) = \Pi_{m=1}^K (X_{im} \le x_m)$.

some $\theta_0 \in \Theta$. Under the alternative hypothesis, however, we have $E[g(W_i, \theta, x) \neq 0$ for some $x \in R^K$, for all $\theta \in \Theta$. This suggests that we can consider the following Kolmogorov–Smirnov and Cramer–von Mises type tests statistics:

$$KS_n = \sup_{x \in R^K} |\nu_n(x, \hat{\theta})| \,;\, CM_n = \frac{1}{n}\sum_{i=1}^{n} \nu_n^2(X_i, \hat{\theta}) \tag{3}$$

where $\nu_n(x, \theta) = (1/\sqrt{n})\Sigma_{i=1}^n g(W_i, \theta, x)$ and $\hat{\theta}$ is an estimator of θ_0.

Let wp1 denote 'with probability 1.' To derive the asymptotic null distributions, we need the following assumptions:

Assumption A1. $\{(Y_i, X_i) : i \geq 1\}$ are *i.i.d.* with joint distribution function $H(y, x)$ of (Y_i, X_i), conditional distribution function $H(\cdot|x)$ of Y_i given $X_i = x$ and marginal distribution function $G(\cdot)$ of X_i.

Assumption A2. (i) $\rho(Y_i, \theta)$ is differentiable in θ on a neighborhood N_1 of θ_0 $\forall i \geq 1$. (ii) $\sup_{x \in R^K}$ $\sup_{\theta:\|\theta-\theta_0\| \leq r_n} \|1/n\Sigma_{i=1}^n\ \partial/\partial\theta\ \rho(Y_i, \theta)(X_i \leq x) - \Delta_0(x)\| \to 0$ wpl for all sequences of positive constants $\{r_n : n \geq 1\}$ such that $r_n \to \theta$, where $\Delta_0(x) = E\partial/\partial\theta\rho(Y_i, \theta_0)(X_i \leq x)$. (iii) $\sup_{x \in R^k}\|\Delta_0(x)\| < \infty$ and $\Delta_0(\cdot)$ is uniformly continuous on R^K with respect to the metric $l(x_1, x_2) = [E(\rho(Y_i,\theta_0)\ ((X_i \leq x_1) - (X_i \leq x_2)))^2]^{1/2}$. (iv) $E|\rho(Y_i, \theta_0)|^{2+\delta} < \infty$ for some $\delta > \theta$.

Assumption A3. $\sqrt{n}(\hat{\theta} - \theta_0) = D_0 1/\sqrt{n}\Sigma_{i=1}^n \psi(Y_i, X_i, \theta_0) + o_p(1)$, where D_0 is a non-random $P \times P$ matrix that may depend on θ_0, $\psi(\cdot,\cdot,\cdot) : R^L \times R^K \times \Theta \to R^P$ is a measurable function that satisfies $E\psi(Y_i, X_i, \theta_0) = 0$ and $E\|\psi(Y_i, X_i, \theta_0)\|^{2+\varepsilon} < \infty$ for some $\varepsilon > \theta$.

Assumption A3 is very general and can be verified for many estimators that are $\sqrt{n}$-consistent and asymptotically normal using the results in the literature. For example, consider the moment condition $Ef(W_i, \theta_0) = 0$ which holds under our null hypothesis, where $f(W_i, \theta) = \rho(Y_i, \theta)A(X_i)$ and $A(\cdot) : R^K \to R^M$ is a measurable function. Let $\bar{f}(\theta) = 1/n\Sigma f(W_i, \theta)$. If $\hat{\theta}$ is the one-step GMM estimator that minimizes $\bar{f}(\theta)'\Omega\bar{f}(\theta)$ for some fixed matrix Ω, then, Assumption A3 holds under general conditions with $D_0\ (J_0'\Omega J_0')^{-1}\ J_0'\Omega$ and $\psi(Y_i, X_i, \theta_0) = f(W_i, \theta_0)$, where $J_0 = E(\partial f(W_i, \theta_0)/\partial\theta')$. On the other hand, if $\hat{\theta}$ is the two-step efficient GMM estimator minimizing $\bar{f}(\theta)'\Omega_n(\tilde{\theta})\bar{f}(\theta)$ where $\Omega_n(\theta) = 1/n\Sigma f(W_i, \theta)f(W_i, \theta)']^{-1}$ and $\tilde{\theta}$ is the one-step estimator, then Assumption A3 holds with $D_0 = (J_0'\Omega_0 J_0')^{-1}\ J_0'\Omega_0$ and $\psi(Y_i, X_i, \theta_0) = f(W_i, \theta_0)$, where $\Omega_0[Ef(W_i, \theta_0)f(W_i, \theta_0)']^{-1}$.

Now we show that the asymptotic null distributions of our test statistics are functionals of a mean zero Gaussian process $(\nu(\cdot), \nu_0')'$ with covariance function given by $C(x_1, x_2, \theta_0, H)$, where

$$C(x_1, x_2, \theta, H) = \int\int \begin{pmatrix} p(y,\theta)(x \leq x_1) \\ \psi(y, x, \theta) \end{pmatrix} \begin{pmatrix} \rho(y, \theta)(x \leq x_2) \\ \psi(y, x, \theta) \end{pmatrix}' \mathrm{d}H(y, x). \tag{4}$$

Theorem 1. *If Assumptions A1–A3 hold under the null, then*

$$(a) \quad KS_n \xrightarrow{d} \sup_{x \in R^K} |\nu(x) + \Delta_0(x)' D_0 \nu_0|;$$

$$(b) \quad CM_n \xrightarrow{d} \int (\nu(x) + \Delta_0(x)'D_0\nu_0)^2 \, dG(x).$$

The asymptotic null distributions of KS_n and CM_n depend on the 'true' parameter θ_0 and distribution function $G(\cdot)$. The latter implies that the asymptotic critical values for KS_n and CM_n cannot be tabulated. However, the bootstrap procedure can be used to approximate the null distributions.

3. Bootstrap approximation

The basic problem for bootstrapping a test statistic is how to impose the null hypothesis in the resampling scheme. That is, the essential problem is to find a bootstrap distribution that mimics the null distribution of the test statistic, even though the data fails to satisfy the null hypothesis.

Let $\{W_i^* = (Y_i^{*\prime}, X_i^{*\prime})': i \le n\}$ denote a bootstrap sample randomly drawn (with replacement) from the empirical distribution of $\{W_i : i \le n\}$. Let E^* denote the expectation relative to the distribution of the bootstrap sample conditional on the original sample. Define $g^*(w, \theta, \cdot) = \rho(y, \theta)(x \le \cdot) - E^*g(W_i, \hat{\theta}, \cdot)$. Note that $E^*[g^*(W_i^*, \hat{\theta}, x) = 0$. The latter can be considered as a bootstrap version of the moment condition $E[g(W_i, \theta_0, x)] = 0$. By recentering, therefore, the bootstrap version satisfies the null restriction[3]. Now, compute the test statistics using the bootstrap sample:

$$KS_n^* = \sup_{x \in R^K} |\nu_n^*(x, \hat{\theta}^*)|; \; CM_n^* = \frac{1}{n}\sum_{i=1}^{n} \nu_n^2(X_i^*, \hat{\theta}^*), \tag{5}$$

where $\nu_n^*(x, \theta) = (1/\sqrt{n}) \sum_{i=1}^n g^*(W_i^*, \theta, x)$, and $\hat{\theta}^*$ is an estimator of $\hat{\theta}$. If we repeat this step B times, then we can get the bootstrap distributions of our test statistics. Let $c_{\alpha nB}^{KS}(\hat{\theta})$ (or $c_{\alpha nB}^{CM}(\hat{\theta})$ be the $(1-\alpha)$-th sample quantile of the bootstrap distribution of KS_i^* (or CM_n^*). It is called the *bootstrap critical value* of significance level α. Thus, we reject the null hypothesis at the significance level α if $KS_n > c_{\alpha nB}^{KS}(\hat{\theta})$ (or $CM_n > c_{\alpha nB}^{CM}(\hat{\theta})$). To justify this procedure, we need the following assumptions:

Assumption B1. (i) $\sqrt{n}(\hat{\theta}^* - \hat{\theta}) = D_0 1/\sqrt{n} \sum_{i=1}^n (Y_i^*, X_i^*, \hat{\theta}) + o_p(1)$ conditional on $(\mathbb{Y}, \mathbb{X}) = \{(Y_i, X_i) : i \le 1\}$ wp1 for D_0 as defined in Assumption A3 (i) and $E^*\psi(Y_i^*, X_i^*, \hat{\theta}) = 0$. (ii) $E \sup_{\theta \in N_1} \|\psi(Y_i, X_i, \theta)\|^{2+\varepsilon}$ for some $\varepsilon > 0$. (iii) $E \sup_{\theta \in N_1} |\rho(Y_i, \theta)|^{2+\delta} < \infty$ and $E \sup_{\theta \in N_1} \|\partial/\partial\theta\rho(Y_i, \theta)\| 2 + \delta/1 + \delta < \infty$ for some $\delta > \theta$.

Assumption B2. $C(x_1, x_2, \theta, H)$ is continuous in θ at θ_0 $\forall x_1, x_2 \in R^K$.

Assumption B3. $\hat{\theta} \to \theta_1$ a.s. for some $\theta_1 \in \Theta$.

We now establish that the bootstrap distribution of KS_n^* (CM_n^*) has the same limit as the asymptotic null distribution of KS_n (CM_n):

Theorem 2. *Suppose Assumptions* A1, A2, B1, B2 *and* B3 (*with* $\theta_1 = \theta_0$) *hold. Then, we have* (*a*)

[3] The idea of recentering a criterion function to mimic a population moment condition is not new in the bootstrap literature, see, e.g. Hall and Horowitz (1996).

$KS_n^* \xrightarrow{d} \sup_{x \in R^K} |\nu(x) + \Delta_0(x)' D_0 \nu_0|$ conditional on $(\mathbb{Y}, \mathbb{X})$ $wp1$ and (b) $CM_n^* \xrightarrow{d} \int (\nu(x) + \Delta_0(x)' D_0 \nu_0)^2$ $\mathrm{d}G(x)$ conditional on $(\mathbb{Y}, \mathbb{X})$ $wp1$.

Note that the limit distribution is absolutely continuous because it is the supremum (or integral) of a Gaussian process with nonsingular covariance matrix. Therefore, Theorem 2 implies that the level α critical value $c_{\alpha nB}^{KS}(\hat{\theta})$(or $c_{\alpha nB}^{CM}(\hat{\theta})$) obtained from the bootstrap distribution of KS_n^* (or CM_n^*) converges (conditional on the original sample) to the critical value $c_\alpha^{KS}(\theta_0)$(or $c_\alpha^{KS}(\theta)$) from the limit distribution of KS_n (or CM_n) as $B \to \infty$ and $\to \infty$. This suggests that the asymptotic significance level of our tests using the bootstrap critical values is α as desired.[4]

4. Power properties

In this section we discuss the power properties of our tests against a sequence of local alternatives and a fixed alternative.

First, consider the local alternatives of the following form:

$$H_A : E\,[\rho(Y_i, \theta_0)|X_i] = d(X_i)/\sqrt{n} \text{ a.s. for } n = 1,2, \ldots, \tag{6}$$

where $d(\cdot)$ is a non-zero function with $E|d(X_i)| < \infty$. Under H_A, we assume that $\{(Y_i, X_i) : i \leq n\}$ are independently distributed with the conditional distribution of $Y_i|X_i$ given by $Q_n(\cdot|X_i)$ and $X_i \sim G(\cdot)$. We further assume

Assumption C1. $\sqrt{n}(\hat{\theta} - \theta_0) = D_0 \frac{1}{\sqrt{n}} \Sigma_{i=1}^n \psi(Y_i, X_i, \theta_0) + o_p(1)$, where D_θ is a non-random $P \times P$ matrix that may depend on θ_0, $\psi(\cdot,\cdot,\cdot) : R^L \times R^K \times \Theta \to R^P$ is a measurable function that satisfies (i) $\sqrt{n} \int\int \psi(y, x, \theta_0) \mathrm{d}Q_n(y|x) \mathrm{d}G(x) \to \xi(\theta_0)$ as $n \to \infty$ for some $\xi(\cdot) : \Theta \to R^P$, (ii) $\sup_{n \geq 1} \int\int \|\psi(y, x, \theta)\|^{2+\varepsilon} \mathrm{d}Q_n(y|x) \mathrm{d}G(x) < \infty$ for some $\varepsilon > 0$ and (iii)$\int\int \psi^2(y, x, \theta_0) \mathrm{d}(Q_n(y|x) - H(y|x)) \mathrm{d}G(x) \to 0$ as $n \to \infty$.

Assumption C2. $\sup_{x^* \in R^K} \int\int \rho^2(y, \theta_0)(x \leq x^*) \mathrm{d}(Q_n)(y|x) - H(y|x)) \mathrm{d}G(x) \to 0$

Let

$$M_1 = \sup_{x \in R^K} |\nu(x) + \Delta_0(x)' D_0 \nu_0 + \delta(x)| \text{ and} \tag{7}$$

$$M_2 = \int \nu(x) + \Delta_0(x)' D_0 \nu_0 + \delta(x))^2 \, \mathrm{d}G(x), \text{ where} \tag{8}$$

$$\delta(x) = \int \mathrm{d}(\tilde{x})(\tilde{x} \leq x) \mathrm{d}G(\tilde{x}) + \Delta_0(x)' D_0 \xi(\theta_0)$$

The asymptotic distributions of KS_n and CM_n under the local alternatives are given by:

[4] For a more detailed discussion, see Whang (1998) Corollary 3.

Theorem 3. *If Assumptions A2, C1, and C2 hold under H_A, then we have (a) $KS_n \xrightarrow{d} M_1$ and (b) $CM_n \xrightarrow{d} M_2$.*

Theorem 3 implies that our tests have non-trivial power against a sequence of $1/\sqrt{n}$ local alternatives and the asymptotic local powers of the tests KS_n and CM_n are given by $P(M_1 > c_\alpha^{KS}(\theta_0))$ and $P(M_2 > c_\alpha^{CM}(\theta_0))$ respectively.

Finally, we show that the tests KS_n and CM_n are consistent against the fixed alternative H_1:

Theorem 4. *Suppose Assumptions A1, A2 and B3 hold and $E|\rho(Y_i, \theta_i)| < \infty$ under the alternative hypothesis H1. Then, for all sequences of rv's $\{c_n : n \geq 1\}$ with $c_n = O_p(1)$, we have $\lim_{n\to\infty} P(KS_n > c_n) = 1$ and $\lim_{n\to\infty} P(CM_n > c_n) = 1$.*

Since the bootstrap critical value $c_{\alpha nB}(\hat{\theta})$ converges to $c_\alpha(\theta_1) < \infty$ wp1 under H_1, it satisfies the requirement of Theorem 4 on c_n.

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Appendix A

The asymptotic results in the main text can be proved using the approach of Andrews (1997) and Whang (1998). We just sketch the main arguments. The proof of Theorem 1 is similar to that of Theorem 2 and hence is omitted.

Proof of Theorem 2. Conditional on $(\mathbb{Y}, \mathbb{X})$, we have

$$\begin{aligned}
\nu_n^*(x, \hat{\theta}^*) &= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}[\rho(Y_i^*, \hat{\theta}^*)(X_i^* \leq x) - E^*\rho(Y_i^*, \hat{\theta})(X_i^* \leq x)] \\
&= \nu_n^*(x, \hat{\theta}) + \left[\frac{1}{n}\sum_{i=1}^{n}\frac{\partial}{\partial\theta'}\rho(Y_i^*, \hat{\theta}_n^*(x))(X_i^* \leq x)\right]\sqrt{n}(\hat{\theta}^* - \hat{\theta}) \\
&= \nu_n^*(x, \hat{\theta}) + \Delta_0(x)'D_0\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\psi(Y_i^*, X_i^*, \hat{\theta})\, o_p(1)
\end{aligned} \tag{A.1}$$

uniformly in x, where the first equality holds by a mean value expansion with $\hat{\theta}_n^*(x) \in (\hat{\theta}^* \hat{\theta})$ and the second equality follows from Assumptions A2 and B1. By the functional CLT of Pollard (1990), (Theorems 10.2 and 10.6) and Lemma A.3 of Whang (1998), we also have

$$\left(\nu_n^*(\cdot, \hat{\theta})\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\psi(Y_i^*, X_i^*, \hat{\theta})'\right)' \Rightarrow (\nu(\cdot), \nu_0')', \tag{A.2}$$

conditional on $(\mathbb{Y}, \mathbb{X})$ wp1, where $\Rightarrow$ denotes the weak convergence. Now, Theorem 2 holds by the continuous mapping theorem (see Pollard (1984, Theorem IV.12, p. 70)). □

Proof of Theorem 3. Under the local alternatives, a mean value expansion argument gives

$$\nu_n(x, \hat{\theta}) = \nu_n(x, \theta_0) + \Delta_0(x)' D_0 \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \psi(Y_i, X_i, \theta_0) + o_p(1), \text{ uniformly in } x. \tag{A.3}$$

Note

$$\left(\nu_n(\cdot, \theta) \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \psi(Y_i, X_i, \theta_0)' \right)' \Rightarrow \left(\nu(\cdot) + \int \mathrm{d}(x)(x \leq \cdot)\, \mathrm{d}G(x), \nu_0' + \xi(\theta_0)' \right) \tag{A.4}$$

by the functional CLT of Pollard. Theorem 3 now follows from the continuous mapping theorem.

Proof of Theorem 4. Let

$$J_n(x, \theta) = \frac{1}{n} \sum^{n} \rho(Y_i, \theta)(X_i \leq x); \; J(x, \theta) = \int \rho(y, \theta)(\tilde{x} \leq x)\, \mathrm{d}H(y, \tilde{x}).$$

We have

$$\sup_{x \in R^K} \left| J_n(x, \hat{\theta}) - J_n(x, \theta_1) \right| = o_p(1); \tag{A.5}$$

$$\sup_{x \in R^K} \left| J_n(x, \theta_1) - J_n(x, \theta_1) \right| = o_p(1); \tag{A.6}$$

where (A.5) holds by a mean value expansion argument and (A.6) holds by Lemma A.3 of Whang (1998). Therefore, we have

$$\frac{1}{\sqrt{n}} KS_n = \sup_{x \in R^K} \left| J(x, \theta_1) \right| + o_p(1);$$

$$\frac{1}{n} CM_n = \int (J(x, \theta_1))^2\, \mathrm{d}G(x) + o_p(1).$$

Now, Theorem 4 follows immediately since, under the alternative hypothesis H_1, there exists some $x_0 \in R^K$ for which $J(x_0, \theta_i) \neq \theta$. □

References

Ait-Sahalia, Y., Bickel, P.J., Stoker, T.M., 2000. In: Goodness-of-fit Tests For Kernel Regression With an Application To Option-implied volatilities, unpublished manuscript.

Andrews, D.W.K., 1997. A conditional Kolmogorov test. Econometrica 65, 1097–1128.

Bierens, H.J., 1982. Consistent model specification tests. Journal of Econometrics 20, 105–134.

Bierens, H.J., 1990. A consistent conditional moment test of functional form. Econometrica 58, 1443–1458.

Bierens, H.J., Ploberger, W., 1997. Asymptotic theory of integrated conditional moment tests. Econometrica 65, 1129–1151.

De Jong, R.M., 1996. The Bierens test under data dependence. Journal of Econometrics 72, 1–32.

Eubank, R., Spiegelman, S., 1990. Testing the goodness of fit of a linear model via nonparametric regression techniques. Journal of the American Statistical Association 85, 387–392.
Fan, Y., Li, Q., 1996. Consistent model specification tests: omitted variables and semiparametric functional forms. Econometrica 64, 865–890.
Gozalo, P.L., 1993. A consistent model specification test for nonparametric estimation of regression function models. Econometric Theory 9, 451–477.
Hall, P., Horowitz, J.L., 1996. Bootstrap critical values for tests based on generalized-method-of-moments estimators. Econometrica 64, 891–916.
Hansen, L.P., Singleton, K., 1982. Generalized instrumental variable estimation of nonlinear rational expectations models. Econometrica 50, 1269–1286.
Härdle, W., Mammen, E., 1993. Comparing nonparametric versus parametric regression fits. Annals of Statistics 21, 1926–1947.
Hong, Y., White, H., 1996. Consistent specification testing via nonparametric series regressions. Econometrica 63, 1133–1160.
Lewbel, A., 1995. Consistent nonparametric hypothesis tests with an application to Slutsky symmetry. Journal of Econometrics 67, 379–401.
Li, Q., Wang, S., 1998. A simple consistent booststrap test for a parametric regression function. Journal of Econometrics 87, 145–165.
Newey, W.K., 1990. Efficient instrumental variables estimation of nonlinear models. Econometrica 58, 809–837.
Pollard, D., 1984. Convergence of Stochastic Processes. Springer Verlag, New York.
Pollard, D., 1990. Empirical processes: theory and applications. In: CBMS Conference Series in Probability and Statistics, Vol. 2. Institute of Mathematical Statistics, Hayward.
Whang, Y.-J., 1998. In: A Consistent Kolmogorov Test of Parametric Regression Function. Department of economics, Ewha university, Unpublished manuscript.
Whang, Y.-J., 2000. Consistent bootstrap tests of parametric regression functions. Journal of Econometrics 98, 27–46.
Wooldridge, J.M., 1992. A test for functional form against nonparametric alternatives. Econometric Theory 8, 452–475.
Yatchew, A.J., 1992. Nonparametric regression tests based on least squares. Econometric Theory 8, 435–451.
Zheng, J.X., 1996. A consistent test of functional form via nonparametric estimation techniques. Journal of Econometrics 75, 263–289.